Rapidity Dependence of Thermal Parameters

in

Relativistic Heavy Ion Collisions

at RHIC.

J. Strümpfer

Department of Physics, University of Cape Town

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1 Introduction

This project presents the determinations of the rapidity dependence of the thermal parameters (T, μ_S, μ_B, μ_Q). We start by using a statistical thermal model [1] that has been extensively used in reconstructing and predicting particle yields from relativistic heavy ion collisions, it has provided us with the interdepence of the thermal parameters and has been verified to give good results [2]. We use the data obtained at the Relativistic Heavy Ion Collider (RHIC) at the Brookhaven National Laboratory. The data was taken from the gold on gold collisions in the Broad RAnge Hadron Magnetic Spectrometers (BRAHMS) experiment [3] at $\sqrt{s_{NN}} = 200$ GeV. We fit our model to these data to determine the temperature and chemical potentials' dependence on the rapidity of the fireball. In particular, we use numerical integration of momentum distributions to calculate the rapidity distributions instead of using Monte Carlo simulations.

The situation that we are working with is the head to head collision of two nuclei; Au - Au in our case. We assume that there is a perfect overlap of the two nuclei to simplify the analysis. Since the nuclei have been accelerated to 200 GeV (as seen in the centre of mass frame) they are travelling at speeds very close to c ($v \approx 0.99999c$). Once these Lorentz contracted nuclei collide they are excited and the nucleons start interacting.

As the fireball starts expanding and cooling hadrons are produced. Initially inelastic collisions will occur between the formed hadrons until the system cools sufficiently for the inelastic cross-sections to become negligible. This is known as chemical freeze-out. From there elastic collisions will continue, redistributing the particles, until the system has cooled far enough for this to cease, this is known as thermal freeze-out. Once both of these have occured chemical equilibrium and thermal equilibrium is reached. The situation after freeze-out (i.e. at equilibrium) is then analyzed using the statistical thermal model.

2 Model

The thermal model describes the system of decaying and interacting hadrons. It has been used to describe the abundances of different particles as well as their momemtum distributions and rapidity distributions. The model thermal model that we employ follows similar assumptions to the Broniowski-Florkowski model. The only difference being the treatment of the expansion of the fireball. These assumptions are described below.

It is an assumption of the model that chemical and thermal freeze-out occur at the same temperature. What this means is that the system stops interacting completely and has reached equilibrium at a single temperature T. Broniowski et al. argued that there is a very short time between the two freeze-outs (≈ 4 fm/c) [4].

The model has built into it also the full treatment of particles arising as decay products from multiple-body decays. This is done following the procedures first done by Schnedermann et al. in [5] and furthered by Yu et al in [6].The PDG datebase [7] is used to compute the abundancies of particles arising from the two and three body decays listed. These calculations are described in the sections below. In this project we have omitted the cascading decays (sequential decays) and resonance decays into four or more particles. This can be done since there are very few of them and they contribute very little to the rapidity spectra.

In general the model also includes the expansion of the fireball, which is known as the flow. This comes from the large momenta of the colliding nuclei that gets transferred to the hadrons in the fireball. In this project the flow is approximated by a distribution of superimposed fireballs at different rapidities. This distribution of fireballs is assumed to have a gaussian shape from the shape of the rapidity spectra produced at RHIC. It is decribed in more detail in sections below.

2.1 Momentum Distributions

The statistical model gives us that the number of particles for a species i is given as:

$$N_i = g_i V \int \frac{d^3 p_i}{(2\pi)^3} f_i(p) \tag{1}$$

Where $f_i(p)$ is the distribution function relevant to the statistics (i.e. Boltzmann, Bose-Einstein or Fermi-Dirac), g_i is the degeneracy factor, V the volume and p_i the 3-momentum. From this model we get that the Lorentz invariant momentum spectra of particle i (using Boltzmann statistics) is given as:

$$E_i \frac{dN_i}{d^3p} = \frac{g_i V}{(2\pi)^3} E_i e^{-E_i/T} e^{\mu_i/T} \tag{2}$$

This distribution, however, only represents the distribution of the 'primordial' particles, i.e. not those that have resulted from decays of resonances. Thus the inclusion of resonance decays must also be made. This inclusion contributes significantly to the model as it can be seen from [4] that only about a quarter of the pion yield in the experiment is accounted for from the 'primordial' distribution. Following from the work of [8] and [6] we can include the momentum distributions coming from two and three body resonance decays. From their results we made an assumption of zero particle widths, which allowed us to proceed slightly further than what they have in integrating the momentum distributions. We proceed by giving a derivation of their results as they were used in our model.

2.1.1 Two Body Decays

We first take a look at the case of two body decays using an example of the Delta decaying into a pion and a nucleon:

$$\Delta \rightarrow \pi + N. \tag{3}$$

We can derive the following relation between the 4-momenta of the particles:

$$p^\mu_\Delta = p^\mu_\pi + p^\mu_N, \tag{4}$$

$$p^\mu_\Delta - p^\mu_\pi = p^\mu_N, \tag{5}$$

$$(p_\Delta - p_\pi)^2 = {p_N}^2, \tag{6}$$

$${p_\Delta}^2 - 2p_\pi p_\Delta + {p_\pi}^2 = {p_N}^2. \tag{7}$$

If the Δ was at rest ($\vec{p}_\Delta = \vec{0}$) then we get that:

$$m^2_\Delta - 2E_0 m_\Delta + m^2_\pi = m^2_N. \tag{8}$$

Then by rearranging the above equation we can solve for the pion energy, E_0, from a stationary Δ decay:

$$E_0 = \frac{m^2_\Delta + m^2_\pi - m^2_N}{2m_\Delta}. \tag{9}$$

If Δ is decaying then we are left with one nucleon and one pion. Thus the number of pions given by N_π must be

$$N_\pi = \int \frac{d^3 p_\pi}{E_\pi} \left(E_\pi \frac{dN_\pi}{d^3 p_\pi} \right) = 1. \tag{10}$$

From this equation we can solve for the momentum distribution, and we know from equation (9) that the energy of pion in this distribution must be given by E_0. Thus:

$$E_\pi \frac{dN_\pi}{d^3 p_\pi} = \frac{\delta \left(E_\pi - E_0 \right)}{4\pi p_0}. \tag{11}$$

The momentum of the pion p_0 is fixed by the energy E_0:

$$p_0 \;=\; \sqrt{E_0^2 - m_\pi^2}. \tag{12}$$

Equation (11) corresponds to the momentum distribution of π coming from Δ decays. For the total momentum distribution we must include the distribution of Δ's. The distribution of Δ's is given by equation (1). Multiplying this in to equation (11) we get:

$$E_\pi \frac{dN_\pi}{d^3 p_\pi} = g_\Delta V \int \frac{d^3 p_\Delta}{(2\pi)^3} \, e^{-(E_\Delta - \mu)/T} \, \frac{\delta \left(E_\pi - E_0 \right)}{4\pi p_0}. \tag{13}$$

If the Δ is not at rest then the energy of the pion (E_π) must be boosted. Following similar steps as in equations (4) to (9) it becomes:

$$E_\pi = \frac{{p_\Delta}^\mu \cdot {p_\pi}^\mu}{m_\Delta} = \frac{\vec{p_\Delta} \cdot \vec{p_\pi}}{m_\Delta} - \frac{E_\Delta E_\pi}{m_\Delta}. \tag{14}$$

Substituting this into equation (13) gives us the momentum distribution of pions coming from delta decays:

$$E_\pi \frac{dN_\pi}{d^3 p_\pi} = g_\Delta V \int \frac{d^3 p_\Delta}{(2\pi)^3} \, e^{-(E_\Delta - \mu)/T} \, \frac{\delta \left(\frac{p_\Delta \cdot p_\pi}{m_\Delta} - E_0 \right)}{4\pi p_0}. \tag{15}$$

With a bit of work this can be integrated. First we change from cartesian to spherical co-ordinates:

$$d^3 p_\Delta = {p_\Delta}^2 dp_\Delta d(\cos\theta) d\phi. \tag{16}$$

We also note that from $E_\Delta = \sqrt{p_\Delta{}^2 + m_\Delta{}^2}$ we get that $p_\Delta dp_\Delta = E_\Delta dE_\Delta$. Thus the change to spherical co-ordinates uses

$$d^3 p_\Delta = p_\Delta E_\Delta dE_\Delta d(\cos\theta) d\phi. \tag{17}$$

Using this in equation (15) we can integrate with respect to the azimuthal angle to get the momentum distribution as:

$$\begin{aligned} E_\pi \frac{dN_\pi}{d^3 p_\pi} &= \frac{g_\Delta V}{(2\pi)^3} \frac{1}{4\pi} 2\pi \int \frac{p_\Delta E_\Delta dE_\Delta}{p_0} e^{-(E_\Delta - \mu)/T} \frac{m_\Delta}{p_\Delta p_\pi} \\ &\times \int_{-1}^{+1} \delta \left[\cos\theta - \frac{E_\Delta E_\pi}{p_\Delta p_\pi} + \frac{m_\Delta E_0}{p_\Delta p_\pi} \right] d(\cos\theta). \end{aligned} \tag{18}$$

To proceed from here we need to use the relation $\int dx \delta(x-a) f(x) = f(a)$. We can then integrate with respect to $\cos(\theta)$ to get:

$$E_\pi \frac{dN_\pi}{d^3 p_\pi} = \frac{g_\Delta V}{(2\pi)^3} \frac{1}{2} \int E_\Delta dE_\Delta e^{-(E_\Delta - \mu)/T} \frac{m_\Delta}{p_0 p_\pi}. \tag{19}$$

It is important to note that this expression must still be evaluated between the limits of $\cos\theta = \pm 1$; this will be done shortly. First we carry out the remaining integral over E_Δ to get:

$$E_\pi \frac{dN_\pi}{d^3 p_\pi} = \frac{g_\Delta V}{(2\pi)^3} \frac{1}{2} \frac{m_\Delta}{p_0 p_\pi} \left[-E_\Delta T e^{-(E_\Delta - \mu)/T} - T^2 e^{-(E_\Delta - \mu)/T} \right]_{\cos\theta = -1}^{\cos\theta = +1}. \tag{20}$$

The limits on $\cos\theta$ give us the following:

$$\begin{gathered} -1 < \cos\theta < 1, \\ -1 < \tfrac{E_\Delta E_\pi}{p_\Delta p_\pi} - \tfrac{m_\Delta E_0}{p_\Delta p_\pi} < 1, \\ - \pm p_\Delta p_\pi < E_\Delta E_\pi - m_\Delta E_0 < \pm p_\Delta p_\pi \\ -p_\Delta{}^2 p_\pi{}^2 < E_\Delta{}^2 E_\pi{}^2 - 2 E_\Delta E_\pi E_0 m_\Delta + m_\Delta{}^2 E_0{}^2 < p_\Delta{}^2 p_\pi{}^2. \end{gathered} \tag{21}$$

but $E_\Delta = \sqrt{p_\Delta{}^2 + m_\Delta{}^2}$ and thus we get the following equation from which we can calculate the limits of the integration:

$$E_\Delta{}^2 m_\pi{}^2 - 2 E_\Delta E_\pi E_0 m_\Delta + m_\Delta{}^2 \left(E_0{}^2 + p_\pi{}^2 \right) = 0. \tag{22}$$

This is a quadratic equation in E_Δ whose solution is:

$$E_\Delta{}^\pm = \frac{E_\pi E_0 m_\Delta \pm \sqrt{E_\pi^2 E_0^2 m_\Delta^2 - \left(E_0{}^2 + p_\pi{}^2 \right) m_\Delta^2 m_\pi^2}}{2 m_\pi^2}. \tag{23}$$

Equation (20) must be evaluated between ${E_\Delta}^{\pm}$ and multiplied by b, the probability for the Δ to decay into the π and N (i.e. the branching ratio $b_{\Delta\to\pi+N}$).

To generalise this example we give the case of some resonance R decaying as $R \to 1+2$. Hence Δ gets replaced by R, π by 1 and N by 2. The final result for the momentum distribution of two body decays is:

$$E_1 \frac{dN_1}{d^3p_1} = b \frac{g_R}{16\pi^3} \frac{m_R}{p_0 p_1} \left[\left(E_R T + T^2 \right) e^{-E_R/T} e^{\mu/T} \right]_{E_R^+}^{E_R^-} . \tag{24}$$

Where:

$$\begin{aligned} {E_R}^{\pm} &= \frac{E_1 E_0 m_R \pm \sqrt{E_1^2 E_0^2 m_R^2 - \left({E_0}^2 + {p_1}^2 \right) m_R^2 m_1^2}}{2m_1^2}, && (25) \\ E_0 &= \frac{m_R^2 + m_1^2 - m_2^2}{2m_R}, && (26) \\ p_0 &= \sqrt{E_0^2 - m_1^2}. && (27) \end{aligned}$$

This agrees with the results given by J. Kapusta in [8]

2.1.2 Three Body Decays

The three body decays are, unsurprisingly, more complicated. We start by looking at decays into N bodies.

$$R \to 1 + 2 + \cdots + N. \tag{28}$$

For N body decays we have that the momentum distribution is as follows:

$$E_1 \frac{dN}{d^3p_1} = \int \frac{d^3p_R}{E_R} \left(E_R \frac{dN_R}{d^3p_R} \right) E_1 \frac{d\Gamma_1}{d^3p_1}. \tag{29}$$

This generalizes equation (13) of the two body decays to N body decays. Here Γ_1 is the number of 1's coming from R decays and $E_1 \frac{d\Gamma_1}{d^3p_1}$ is the momentum distribution of particle 1 coming from R decay. As before, working with a Boltzmann distribution, we have that:

$$E_R \frac{dN_R}{d^3p_R} = \frac{g_R V}{(2\pi)^3} E_R e^{-E_R/T} e^{\mu/T}. \tag{30}$$

and:

$$2E_1\frac{d\Gamma_1}{d^3p_1} = \frac{\int \prod_{i=2}^{N} \frac{d^3p_i}{2E_i}\ \delta^4\left(p_R - \sum_{j=1}^{N} p_j\right)}{\int \prod_{i=1}^{N} \frac{d^3p_i}{2E_i}\ \delta^4\left(p_R - \sum_{j=1}^{N} p_j\right)}. \tag{31}$$

In the equation the numerator is the same as the denominator, sans the first factor where $i = 1$. Thus we define:

$$R_N({p_R}^2, {p_1}^2, \cdots, {p_N}^2) = \int \prod_{i=1}^{N} \frac{d^3p_i}{2E_i}\ \delta^4\left(p_R - \sum_{j=1}^{N} p_j\right), \tag{32}$$

and:

$$R_{N-1}((p_R - p_1)^2, {p_2}^2, \cdots, {p_N}^2) = \int \prod_{i=2}^{N} \frac{d^3p_i}{2E_i}\ \delta^4\left((p_R - p_1) - \sum_{j=2}^{N} p_j\right). \tag{33}$$

The numerator of equation (31) is then given by equation (33) and the denominator by (32). Combining equations (32) and (33) we get a relation between the numerator and denominator that we will investigate further:

$$R_N({p_R}^2, {p_1}^2, \cdots, {p_N}^2) = \int \frac{d^3p_1}{2E_1} R_{N-1}((p_R - p_1)^2, {p_2}^2, \cdots, {p_N}^2). \tag{34}$$

Thus there is a recurrence relation between R_N and R_{N-1} which we can use to do the integrals in equation (31). We first define:

$$W^2 = (p_R - p_1)^2 = (p_2 + p_3 + \cdots + p_N)^2. \tag{35}$$

Using this in equation (33) gives us:

$$R_{N-1}({p_W}^2, {p_2}^2, \cdots, {p_N}^2) = \int \tfrac{d^3p_2}{2E_2} \cdots \tfrac{d^3p_N}{2E_N} \delta^4(p_W - p_2 - \cdots - p_N). \tag{36}$$

For the case where $N = 3$ we have four particles with 4-momenta: $p_R^\mu,\ p_1^\mu,\ p_2^\mu$, p_3^μ and the above equation becomes:

$$R_2\left({p_R}^2, {p_1}^2, p_W^2\right) = \int \tfrac{d^3p_1}{2E_1} \tfrac{d^3p_W}{2E_W} \delta^4\left(p_R - p_1 - p_W\right). \tag{37}$$

To find the momentum distribution of the decay product 1 we need to solve the integrals in equation (31). To do this we will proceed by deriving a formula for R_N from the two equations above. Using some foresight we solve the following integral:

$$I = \int dW^2 R_2({p_R}^2, {p_2}^2, {p_W}^2) R_{N-1}({p_W}^2, {p_2}^2, \cdots, {p_N}^2),$$

$$\begin{aligned} &= \int dW^2 \int \frac{d^3p_1}{2E_1} \int \frac{d^3p_W}{2E_W} \delta^4\left(p_R - p_1 - p_W\right), \\ &\times \int \frac{d^3p_2}{2E_2} \cdots \int \frac{d^3p_N}{2E_N} \delta^4(p_W - p_2 - \cdots - p_N). \end{aligned} \tag{38}$$

To proceed, we need the following:

$$\begin{aligned} &\int dp_W^0 \delta({p_W}^2 - W^2)\Theta(p_W^0) = \int dp_W^0 \delta\left({p_W^0}^2 - {\vec{p}_W}^{\,2} - W^2\right)\Theta(p_W^0), \\ &= \int dp_W^0 \Theta(p_W^0) \left\{ \frac{1}{2\sqrt{{\vec{p}_W}^{\,2} + W^2}} \left[\delta\left(p_W^0 - \sqrt{{\vec{p}_W}^{\,2} + W^2}\right) + \delta\left(p_W^0 + \sqrt{{\vec{p}_W}^{\,2} + W^2}\right)\right]\right\}, \\ &= \frac{1}{2\sqrt{{\vec{p}_W}^{\,2} + W^2}} = \frac{1}{2E_W}. \end{aligned} \tag{39}$$

Using the above result in equation (38) we proceed to calculate I.

$$\begin{aligned} I &= \int dW^2 \int \frac{d^3p_1}{2E_1} \int d^4p_W \; \delta({p_W}^2 - W^2)\delta^4\left(p_R - p_1 - p_W\right), \\ &\quad \times \int \frac{d^3p_2}{2E_2} \cdots \frac{d^3p_N}{2E_N} \delta^4(p_W - p_2 - \cdots - p_N), \end{aligned} \tag{40}$$

$$\begin{aligned} &= \int dW^2 \int \frac{d^3p_1}{2E_1} \int \frac{d^3p_2}{2E_2} \cdots \frac{d^3p_N}{2E_N}, \\ &\quad \times \delta^4(p_R - p_1 - p_2 - \cdots - p_N)\delta\left((p_R - p_1)^2 - W^2\right), \end{aligned} \tag{41}$$

$$= \int \frac{d^3p_1}{2E_1} \int \frac{d^3p_2}{2E_2} \cdots \int \frac{d^3p_N}{2E_N} \delta^4(p_R - p_1 - p_2 - \cdots - p_N). \tag{42}$$

If we compare the above equation to the equation for R_N (equation (32)), we see that they are identical. Hence the formula for R_N is:

$$R_N\left(p_R^2, p_1^2, \cdots, p_N^2\right) = \int dW^2 R_2(p_R^2, p_1^2, p_W^2) R_{N-1}(p_W^2, p_2^2, \cdots, p_N^2). \tag{43}$$

In the case where $N = 3$ equation (43) becomes:

$$R_3\left(p_R^2, p_1^2, p_2^2, p_3^2\right) = \int dW^2 R_2(p_R^2, p_1^2, p_W^2) R_2(p_W^2, p_2^2, p_3^2). \tag{44}$$

To proceed any further, the equation above needs to be simplified. This is done by calculating the integrals in R_2. We will do this as given in equation (32) for $N = 2$.

$$\begin{aligned} R_2\left(p_R^2, p_1^2, p_2^2\right) &= \int \frac{d^3p_1}{2E_1} \int \frac{d^3p_2}{2E_2} \delta^4\left(p_R - p_1 - p_2\right), \qquad (45) \\ &= \int \frac{d^3p_1}{2E_1} \int d^4p_2 \delta\left(p_2^2 - m_2^2\right), \end{aligned}$$

$$\times\delta^4\left(p_R - p_1 - p_2\right), \tag{46}$$

$$= \int \frac{d^3p_1}{2E_1}\delta^4\left[\left(p_R - p_1\right)^2 - m_2^2\right], \tag{47}$$

$$= \int \frac{d^3p_1}{2E_1}\delta^4\left[m_R^2 + m_1^2 - 2p_R\cdot p_1 - m_2^2\right]. \tag{48}$$

Note that the result is Lorentz invariant, thus we can choose any frame to evaluate the integral. If we choose the rest frame of the decaying resonance R and hence $p_R = 0$.We continue with R_2:

$$\begin{aligned} R_2\left(p_R^2, p_1^2, p_2^2\right) &= \int \frac{d^3p_1}{2E_1}\delta^4\left[m_R^2 + m_1^2 - 2p_R\cdot p_1 - m_2^2\right], &(49)\\ &= \int \frac{dp_1 p_1^2 d\left(\cos\theta\right)}{2E_1}\delta^4\left[m_R^2 + m_1^2 - 2m_R E_1 - m_2^2\right], &(50)\\ &= 4\pi\int \frac{dE_1 E_1 p_1}{2E_1}\frac{1}{2m_R}\delta^4\left[\frac{m_R^2 + m_1^2 - m_2^2}{2m_R} - E_1\right], &(51)\\ &= \frac{\pi|\vec{p}_1|}{m_R}. &(52)\end{aligned}$$

With $|\vec{p}_1|$ being fixed by $E_1 = \left(m_R^2 + m_1^2 - m_2^2\right)/(2m_R)$ and $p_1^2 = E_1^2 - m_1^2$ (similar to the 2-body decays in the previous section). Thus R_2 is as follows:

$$R_2\left(p_R^2, p_1^2, p_2^2\right) = \frac{\pi\sqrt{\left(m_R^2 - m_1^2 - m_2^2\right)^2 - 4m_1 m_2}}{2m_R^2}. \tag{53}$$

We define $\lambda(x, y, z)$ as:

$$\lambda\left(x, y, z\right) = \left(x - y - z\right)^2 - 4yz. \tag{54}$$

Using this function in our equations for R_2 and R_3 we get:

$$R_2\left(p_R^2, p_1^2, p_2^2\right) = \frac{\pi}{2m_R^2}\lambda^{\frac{1}{2}}\left(m_R^2, m_1^2, m_2^2\right), \tag{55}$$

and

$$\begin{aligned} R_3\left(p_R^2, p_1^2, p_2^2, p_3^2\right) &= \frac{\pi^2}{4m_R^2}\int \frac{dW^2}{W^2}\lambda^{\frac{1}{2}}\left(m_R^2, W^2, m_1^2\right), \\ &\times\ \lambda^{\frac{1}{2}}\left(W^2, m_2^2, m_3^2\right). &(56)\end{aligned}$$

It should be noted that R_3 is only dependant on m_R,m_1, m_2 and m_3. These values are all constant for a given decay and thus R_3 is also a constant for any given decay. Using the equations for R_3 and R_2 that we have found we can rewrite equation (31) as:

$$2E_1 \frac{d\Gamma_1}{d^3 p_1} = \frac{R_2\left(\left(p_R - p_1\right)^2, p_2^2, p_3^2\right)}{R_3\left(p_R^2, p_1^2, p_2^2, p_3^2\right)}. \tag{57}$$

This can then be substituted into the equation for the momentum distribution of particle 1 in the decays (equation (29)) to give:

$$\begin{aligned} E_1 \frac{dN}{d^3 p_1} &= \int \frac{d^3 p_R}{E_R} \left(E_R \frac{dN_R}{d^3 p_R}\right) E_1 \frac{R_2\left(\left(p_R - p_1\right)^2, p_2^2, p_3^2\right)}{R_3\left(p_R^2, p_1^2, p_2^2, p_3^2\right)}, & (58) \\ &= \frac{1}{2R_3} \int d^3 p_R \frac{dN_R}{d^3 p_R} \frac{\pi}{2\left(p_R - p_1\right)^2} \lambda^{\frac{1}{2}} \left(\left(p_R - p_1\right)^2, m_2^2, m_3^2\right). & (59) \end{aligned}$$

Substituting $\frac{dN_R}{d^3 p_R}$ and changing to spherical coordinates (as was done in the previous section in equations (15) to (18)), equation (59) becomes:

$$E_1 \frac{dN}{d^3 p_1} = \frac{gV}{\left(2\pi\right)^3} \frac{\pi}{2R_3} \int d\left(\cos\theta\right) d\phi dE_R \frac{p_R E_R}{2x} e^{-E_R/T} e^{\mu/T} \lambda^{\frac{1}{2}} \left(x, m_2^2, m_3^2\right). \tag{60}$$

The integration over ϕ leads to a factor of 2π while the integration of $\cos\theta$ is not so simple. We first begin by changing the integration of $\cos\theta$ to an integration over x, defined to be:

$$x = m_R^2 + m_1^2 - 2E_R E_1 + 2p_R p_1 \cos\theta = \left(p_R - p_1\right)^2,$$
$$d\left(\cos\theta\right) = \frac{dx}{2p_R p_1}.$$

The momentum distribution is thus given by:

$$\begin{aligned} E_1 \frac{dN}{d^3 p_1} &= \frac{gV}{\left(2\pi\right)^2} \frac{\pi}{4R_3} \int dE_R \frac{p_R E_R}{2p_R p_1} e^{-E_R/T} e^{\mu/T}, \\ &\times \int \frac{dx}{x} \left[x^2 - 2x\left(m_2^2 + m_3^2\right) - \left(m_2^2 - m_3^2\right)^2\right]^{\frac{1}{2}}, \\ &= \frac{gV}{32\pi} \frac{1}{R_3} \frac{1}{p_1} \int dE_R E_R e^{-E_R/T} e^{\mu/T} \int \frac{dx}{x} \sqrt{R}. & (61) \end{aligned}$$

where $R = x^2 - 2x\left(m_2^2 + m_3^2\right) - \left(m_2^2 - m_3^2\right)^2$. The final integral that we are left to do takes the form:

$$I = \int \frac{dx}{x} \sqrt{R} = \int d\left(\sqrt{R}\right) - \left(m_2^2 + m_3^2\right) \int \frac{dx}{\sqrt{R}} + \left(m_2^2 - m_3^2\right)^2 \int \frac{dx}{x\sqrt{R}}, \tag{62}$$

and we will proceed to calculate it term by term. To start, R is rewritten in a way that makes it easier to work with:

$$R = (x - x_+)\,(x - x_-)\,. \tag{63}$$

where $x_+ = (m_2 + m_3)^2$ and $x_- = (m_2 - m_3)^2$.
This prompts a brief invesitgation into the limits for x used in the integral. We have two sources for the limits: from $\cos\theta$ and from $R > 0$(which ensures that $\sqrt{R}$ is real). The limits on x that arise from the limits on $\cos\theta$ are calculated below:

$$-1 \leq \cos\theta = \frac{x - m_R^2 - m_1^2 + 2E_R E_1}{2p_R p_1} \leq +1, \tag{64}$$

$$m_R^2 + m_1^2 - 2E_R E_1 - 2p_R p_1 \leq x \leq m_R^2 + m_1^2 - 2E_R E_1 + 2p_R p_1. \tag{65}$$

If the right-hand side inequality is made as small as possible then we get that $x < (m_R - m_1)^2$. The other limit arises from the fact that $x > 0$ and $R > 0$. This would be where $x > x_+$. These sets of limits on x make the calculation of the limits tricky, they can be summarized as:

$$x_+ = \min(\tilde{x}_+, x_{\mathrm{max}}), \tag{66}$$

$$x_- = \max(\tilde{x}_-, x_{\mathrm{min}}), \tag{67}$$

$$x_{\mathrm{max}} = (m_R - m_1)^2, \tag{68}$$

$$x_{\mathrm{min}} = (m_2 + m_3)^2, \tag{69}$$

$$\tilde{x}_\pm = m_R^2 + m_1^2 - 2E_1 E_R \pm 2p_1 p_R. \tag{70}$$

With $x_\pm$ being the limits of the integral given in equation (62).

We split the integral given in equation (62) into I_1, I_2 and I_3 with:

$$I_1 = \int d\left(\sqrt{R}\right), \tag{71}$$

$$I_2 = \int \frac{dx}{\sqrt{R}}, \tag{72}$$

$$I_3 = \int \frac{dx}{x\sqrt{R}}. \tag{73}$$

The calculation of I_1 follows:

$$I_1 = \sqrt{R}|_{x_+} - \sqrt{R}|_{x_-} = \lambda^{\frac{1}{2}}\left(x_+, m_2^2, m_3^2\right) - \lambda^{\frac{1}{2}}\left(x_-, m_2^2, m_3^2\right). \tag{74}$$

We can thus proceed with I_2 and start by expanding R:

$$\begin{aligned}
\int \frac{dx}{\sqrt{R}} &= \int \frac{dx}{\left[x^2 - (x_+ + x_-)\,x + x_+ x_- + \frac{(x_+ + x_-)^2}{4} - \frac{(x_+ + x_-)^2}{4}\right]^{\frac{1}{2}}}, \\
&= \int \frac{dx}{\left\{\left[x - \frac{x_+ + x_-}{2}\right]^2 - \frac{(x_+ + x_-)^2}{4} + x_+ x_-\right\}^{\frac{1}{2}}}, \\
&= \int \frac{dx}{\left\{\left[x - \frac{x_+ + x_-}{2}\right]^2 - \frac{(x_+ - x_-)^2}{4}\right\}^{\frac{1}{2}}}. \qquad (75)
\end{aligned}$$

Here it should be noted that $x_+ - x_- = 4m_2m_3$ and $\frac{x_+ + x_-}{2} = m_2^2 + m_3^2$. Using these in the equation above gives:

$$I_2 = \int \frac{dx}{\left\{\left[x - (m_2^2 + m_3^2)\right]^2 - 4m_2^2m_3^2\right\}^{\frac{1}{2}}}. \qquad (76)$$

Making a change of variable to y where:

$$y = \frac{x - (m_2^2 + m_3^2)}{2m_3m_3}, \qquad dx = 2m_2m_3dy,$$

leads to

$$I_2 = \int \frac{dy}{\{y^2 - 1\}^{\frac{1}{2}}}. \qquad (77)$$

Another change of variable:

$$\begin{aligned}
y = \cosh z, \quad dy = \sinh z dz \qquad &\Rightarrow \frac{dy}{\{y^2 - 1\}^{\frac{1}{2}}} = dz, \\
z = \ln\left(y + \sqrt{y^2 - 1}\right),
\end{aligned}$$

simplifies the integral. The integral I_2 is thus given as:

$$\begin{aligned}
I_2 &= \int dz = z, \\
&= \ln\left\{\frac{x - (m_2^2 + m_3^2)}{2m_2m_3} + \left[\frac{x^2 + (m_2^2 + m_3^2)^2 - 2x\,(m_2^2 + m_3^2) - 4m_2^2m_3^2}{4m_2^2m_3^2}\right]^{\frac{1}{2}}\right\},
\end{aligned}$$

$$= \ln\left\{\frac{x-(m_2^2+m_3^2)}{2m_2m_3}+\left[\frac{x^2-2x\,(m_2^2+m_3^2)+(m_2^2-m_3^2)^2}{4m_2^2m_3^2}\right]^{\frac{1}{2}}\right\}. \tag{78}$$

Which must be evaluated between x_+ and x_- to complete the integral:

$$I_2 = \ln\left[\frac{x_+-(m_2^2+m_3^2)+\lambda^{\frac{1}{2}}\,(x_+,m_2^2,m_3^2)}{x_--(m_2^2+m_3^2)+\lambda^{\frac{1}{2}}\,(x_-,m_2^2,m_3^2)}\right]. \tag{79}$$

We proceed to calculate I_3 then, starting with a change of variable:

$$x = \frac{1}{t}, \qquad \frac{dx}{x} = -\frac{dt}{t}.$$

The integral I_3 thus becomes:

$$\begin{aligned}
I_3 &= -\int \frac{dt}{t}\left[\frac{1}{t^2}-(x_++x_-)\,\frac{1}{t}+x_+x_-\right]^{-\frac{1}{2}},\\
&= -\int dt\left[1-(x_++x_-)\,t+x_+x_-t^2\right]^{-\frac{1}{2}},\\
&= -\frac{1}{\sqrt{x_+x_-}}\int dt\left[t^2-\frac{x_++x_-}{x_+x_-}t+\frac{1}{x_+x_-}\right]^{-\frac{1}{2}},\\
&= -\frac{1}{\sqrt{x_+x_-}}\int dt\left[t^2-\frac{x_++x_-}{x_+x_-}t+\left(\frac{x_++x_-}{2x_+x_-}\right)^2-\left(\frac{x_++x_-}{2x_+x_-}\right)^2\frac{1}{x_+x_-}\right]^{-\frac{1}{2}},\\
&= -\frac{1}{\sqrt{x_+x_-}}\int dt\left[\left(t-\frac{x_++x_-}{2x_+x_-}\right)^2+\frac{1}{x_+x_-}-\left(\frac{x_++x_-}{2x_+x_-}\right)^2\right]^{-\frac{1}{2}}.
\end{aligned} \tag{80}$$

Here we note that:

$$\begin{aligned}
\frac{1}{x_+x_-}-\left(\frac{x_++x_-}{2x_+x_-}\right)^2 &= \frac{4x_+x_--x_+^2-x_-^2-2x_+x_-}{4x_+^2x_-^2},\\
&= \frac{-x_+^2-x_-^2+2x_+x_-}{4x_+^2x_-^2},\\
&= \frac{(x_+-x_-)^2}{4x_+^2x_-^2}.
\end{aligned}$$

Using this in (80) gives:

$$I_3 = -\frac{1}{\sqrt{x_+x_-}}\int dt\left[\left(t-\frac{x_++x_-}{2x_+x_-}\right)^2-\left(\frac{x_+-x_-}{2x_+x_-}\right)^2\right]^{-\frac{1}{2}},$$

$$= \quad -\frac{1}{\sqrt{x_+ x_-}} \left[\frac{x_+ - x_-}{2\,|x_+ x_-|}\right]^{-1} \int dt \left[\frac{\left(t - \frac{x_+ + x_-}{2x_+ x_-}\right)^2}{\left(\frac{x_+ - x_-}{2x_+ x_-}\right)^2} - 1\right]^{-\frac{1}{2}}. \tag{81}$$

Changing variables again to $y = \left(t - \frac{x_+ + x_-}{2x_+ x_-}\right) / \left(\frac{x_+ - x_-}{2x_+ x_-}\right)$ with $dt = dy\frac{x_+ - x_-}{2x_+ x_-}$, will in turn give us I_3 as follows:

$$\begin{aligned} I_3 &= -\frac{1}{\sqrt{x_+ x_-}} \int \frac{dy}{\left[y^2 - 1\right]^{\frac{1}{2}}}, \\ &= -\frac{1}{\sqrt{x_+ x_-}} \ln\left(y + \sqrt{y^2 - 1}\right). \end{aligned} \tag{82}$$

Which must be evaluated between the limits of integration. To do so we first substitute x back into the equation. From the various substitutions and changes of variable that we made we get:

$$\begin{aligned} y &= \frac{\left(m_2^2 - m_3^2\right)^2 - x\left(m_2^2 + m_3^2\right)}{2m_2 m_3 x}, \\ x_+ + x_- &= 2\left(m_2^2 + m_3^2\right), \\ x_+ - x_- &= 4m_2 m_3, \\ x_+ x_- &= \left(m_2^2 - m_3^2\right)^2. \end{aligned}$$

Substituting these gives us that:

$$\begin{aligned} \sqrt{y^2 - 1} &= \left[\frac{\left(m_2^2 - m_3^2\right)^4 - 2x\left(m_2^2 - m_3^2\right)^2\left(m_2^2 + m_3^2\right) + x^2\left(m_2^2 + m_3^2\right)^2 - 4m_2^2 m_3^2 x^2}{4m_2^2 m_3^2 x^2}\right]^{\frac{1}{2}}, \\ &= \left[\frac{\left(m_2^2 - m_3^2\right)^4 - 2x\left(m_2^2 - m_3^2\right)^2\left(m_2^2 + m_3^2\right) + x^2\left(m_2^2 - m_3^2\right)^2}{4m_2^2 m_3^2 x^2}\right]^{\frac{1}{2}}, \\ &= \left|m_2^2 - m_3^2\right| \frac{\left[\left(m_2^2 - m_3^2\right) - 2x\left(m_2^2 + m_3^2\right) + x^2\right]^{\frac{1}{2}}}{2m_2 m_3 x}, \\ &= \left|m_2^2 - m_3^2\right| \frac{\lambda^{\frac{1}{2}}\left(x, m_2^2, m_3^2\right)}{2m_2 m_3 x}. \end{aligned} \tag{83}$$

Using the equations for y and $\sqrt{y^2 - 1}$ in (82) gives us:

$$I_3 \quad = \quad -\frac{1}{\sqrt{x_+ x_-}} \ln\left[\frac{\left(m_2^2 - m_3^2\right)^2 - x\left(m_2^2 + m_3^2\right)}{2m_2 m_3 x} + \left|m_2^2 - m_3^2\right| \frac{\lambda^{\frac{1}{2}}\left(x, m_2^2, m_3^2\right)}{2m_2 m_3 x}\right],$$

$$
\begin{aligned}
&= -\frac{1}{|m_2^2 - m_3^2|} \ln \left[\frac{\left(m_2^2 - m_3^2\right)^2 - x\left(m_2^2 + m_3^2\right) + |m_2^2 - m_3^2| \lambda^{\frac{1}{2}}\left(x, m_2^2, m_3^2\right)}{2 m_2 m_3 x} \right], \\
&= \frac{-\ln\left[\left(m_2^2 - m_3^2\right)^2 - x\left(m_2^2 + m_3^2\right) + |m_2^2 - m_3^2| \lambda^{\frac{1}{2}}\left(x, m_2^2, m_3^2\right)\right]}{|m_2^2 - m_3^2|} \\
&\quad + \frac{\ln[x]}{|m_2^2 - m_3^2|} + \frac{\ln[2 m_2 m_3]}{|m_2^2 - m_3^2|}. \qquad (84)
\end{aligned}
$$

Now that we have I_3 in terms of x we can evaluate it at its limits of integration. This gives:

$$
\begin{aligned}
I_3 &= -\frac{1}{|m_2^2 - m_3^2|} \ln \left[\frac{\left(m_2^2 - m_3^2\right)^2 - x_+\left(m_2^2 + m_3^2\right) + |m_2^2 - m_3^2| \lambda^{\frac{1}{2}}\left(x_+, m_2^2, m_3^2\right)}{\left(m_2^2 - m_3^2\right)^2 - x_-\left(m_2^2 + m_3^2\right) + |m_2^2 - m_3^2| \lambda^{\frac{1}{2}}\left(x_-, m_2^2, m_3^2\right)} \right] \\
&\quad + \frac{1}{|m_2^2 - m_3^2|} \ln \left[\frac{x_+}{x_-} \right] \qquad (85)
\end{aligned}
$$

Using equations (74),(79) and (85) in equation (61) will then give us the final result, which is summarised below.

For a resonance R decaying as R $\rightarrow 1 + 2 + 3$ the momentum distribution is given by:

$$
\begin{aligned}
E_1 \frac{dN_1}{d^3 p_1} &= b \frac{g_R V}{32\pi} \frac{1}{R_3} \frac{1}{p_1} \int_{m_R}^{\infty} e^{-E_R/T} e^{\mu/T} E_R \, dE_R, \\
&\times \left[\left(\sqrt{\lambda^+} - \sqrt{\lambda^-}\right) + \left|m_2^2 - m_3^2\right| \ln \frac{X^+}{X^-} \right. \\
&- \left|m_2^2 - m_3^2\right| \ln \frac{\left(m_2^2 - m_3^2\right)^2 - \left(m_2^2 + m_3^2\right) X^+ + |m_2^2 - m_3^2| \sqrt{\lambda^+}}{\left(m_2^2 - m_3^2\right)^2 - \left(m_2^2 + m_3^2\right) X^- + |m_2^2 - m_3^2| \sqrt{\lambda^-}} \\
&- \left. \left(m_2^2 + m_3^2\right) \ln \frac{-\left(m_2^2 + m_3^2\right) + X^+ + \sqrt{\lambda^+}}{-\left(m_2^2 + m_3^2\right) + X^- + \sqrt{\lambda^-}} \right]. \qquad (86)
\end{aligned}
$$

Where we have that:

$$
\begin{aligned}
\lambda_\pm &= \lambda\left(x_\pm, m_2^2, m_3^2\right) && (87) \\
x_+ &= \min(\tilde{x}_+, x_{\max}), && (88) \\
x_- &= \max(\tilde{x}_-, x_{\min}), && (89) \\
x_{\max} &= (m_R - m_1)^2, && (90) \\
x_{\min} &= (m_2 + m_3)^2, && (91) \\
\tilde{x}_\pm &= m_R^2 + m_1^2 - 2 E_1 E_R \pm 2 p_1 p_R. && (92)
\end{aligned}
$$

To calculate the full momentum distribution for hadron i we thus have to sum over all two and three body decays containing i (the sets J_2 and J_3 respectively).

$$E_i \frac{dN_i^{total}}{d^3p} = E_i \frac{dN_i^{primordial}}{d^3p} + \sum_{j \epsilon J_2} E_j \frac{dN_j^{2-body}}{d^3p} + \sum_{j \epsilon J_3} E_j \frac{dN_j^{3-body}}{d^3p}. \qquad (93)$$

2.2 Rapidity Distributions

The rapidity distribution of the particles $\frac{dN}{dy}(y)$ determine the rapidity dependence of the thermal parameters. The transverse mass m_T is defined as:

$$m_T = \sqrt{m^2 + p_x^2 + p_y^2}. \qquad (94)$$

Using the transverse mass and the rapidity y we give the momentum 4-vector:

$$p^\mu = (m_T \cosh y, p_x, p_y, m_T \sinh y). \qquad (95)$$

Thus we have $E = m_T \cosh y$ which mean that $E\frac{dN}{d^3p} = E\frac{dN}{d^3p}(m_T, y, \mu)$, where μ is the chemical potential. The rapidity y can be calculated from:

$$E = m_T \frac{e^y + e^{-y}}{2}, \qquad (96)$$

$$p_z = m_T \frac{e^y - e^{-y}}{2}, \qquad (97)$$

These two lead to:

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z} \qquad (98)$$

With the understanding of the rapidity dependence on the energy E and three-momentum $\vec{p}$ we can see than the momentum distribution can be written as:

$$E_1 \frac{dN_1}{d^3p} = \frac{dN_1}{dy\, m_T\, dm_T\, d\phi}. \qquad (99)$$

From this we can determine the rapidity distribution:

$$\frac{dN_1}{dy} = \int_0^{2\pi} \int_{m_1}^{\infty} E_1 \frac{dN_1}{d^3p_1} m_T dm_T d\phi \qquad (100)$$

To determine the final rapidity distribution we assume that we have a distribution of fireballs given by $\rho(y_{FB})$ with y_{FB} being the rapidity of the fireball. Taking a lead from experimental results we give this distribution the form of a Gaussian distribution:

$$\rho(y_{FB}) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{y_{FB}^2}{2\sigma^2}\right). \tag{101}$$

The final rapidity ditribution of a hadron i is then calculated by integrating over all possible fireball rapidities:

$$\frac{dN_\rho^i}{dy} = \int_{-\infty}^{\infty} \rho\left(y_{FB}\right) \frac{dN_1^i}{dy}(y - y_{FB})\, dy_{FB}. \tag{102}$$

The width, σ, of the distribution thus also becomes an important parameter to determine.

2.3 Thermal Parameters

The chemical potential μ_i takes care of the conserved quantities in the fireball. Since we are at chemical equilibrium and are interested in strong interaction, the conserved quantities are the quantum numbers for charge, baryon number, strangeness, charm, beauty, etc. In this project only the chemical potentials for baryon number, charge and strangeness are taken into account. Thus the chemical potential μ_i is written as:

$$\mu_i = B_i\mu_B + Q_i\mu_Q + S_i\mu_S. \tag{103}$$

This allows some fluctuation for each of the quantum numbers, but conserves the average quantum numbers. This provides a very good approximation of conserved quantities as the number of particles that we are dealing with is very large. It is these chemical potentials along with the temperature that are thermal parameters we are interested in finding the rapidity dependence for. It is important to note that they are not independent. For each fireball, the initial conditions will fix μ_S, μ_Q and T for a given μ_B. These relations (between μ_B and the other thermal parameters) are already very well known [9]. The relation between T and μ_B is shown in the plot given below.

The relation between the thermal parameters and a parameterisation of the baryon chemical protential as:

$$\mu_B = a + by_{FB}^2, \tag{104}$$

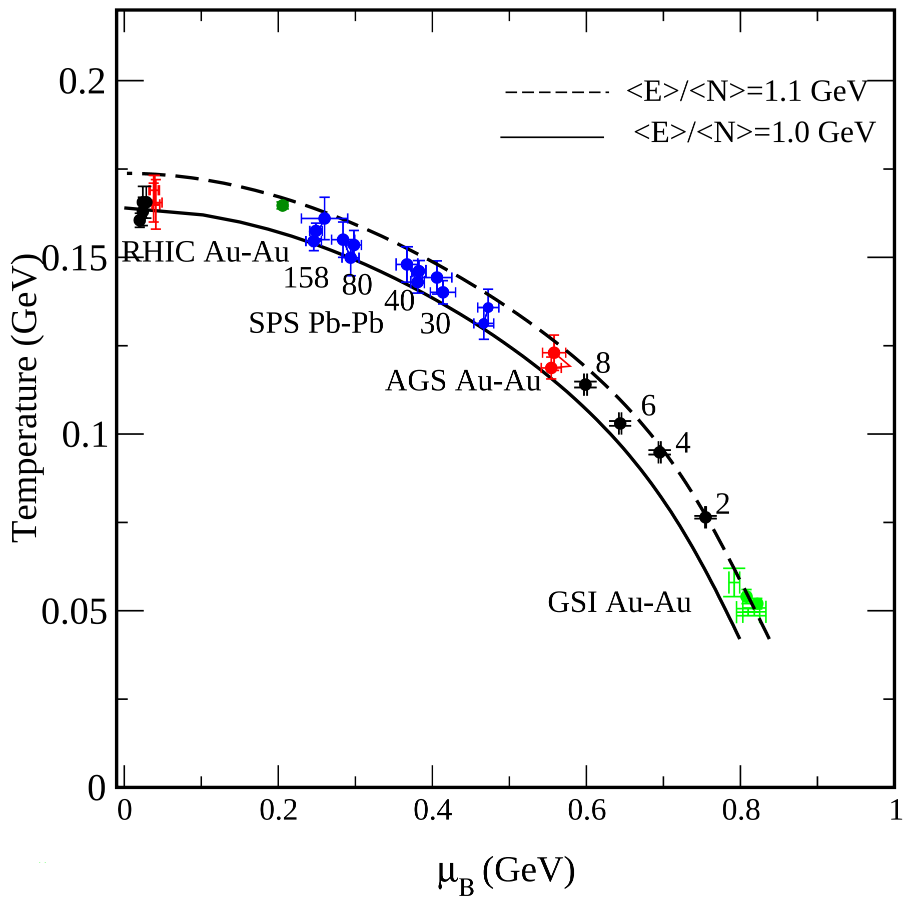

Figure 1: Temperature vs. μ_B us determined from heavy ion collisions at different beam energies from Becattini & Cleymans in [2].

is thus used to calculate, for a given fireball rapidity, μ_B, T, μ_S and μ_Q

3 Method

To calculate the rapidity distribution for a specific hadron, we needed to know in what decays the hadron appears as a decay product. This means that a list of resonance decays are needed along with their branching ratios. A table was compiled from the data provided by the Particle Data Group containing a list of resonances, their decays along with their branching ratios, quantum numbers (Baryon number, Strangeness and Charge), degeneracies and masses. This table then provided the database from which the rapidity distribution of any particular particle could be calculated.

For a given set of parameters σ, a and b and for a given rapidity y the value of $\frac{dN}{dy}$ can be calculated using equation (102). In the integral, the momentum distribution given in equation (93) is used and the fireball rapidity y_{FB} then determines μ_B from equation (104). The temperature and chemical potentials were then determined using the relationships aboce. The integration was done numerically using Gaussian Quadrature with 15 integration points.

The Minuit routine in the ROOT package from CERN was used to do a χ^2 minimization to determine which values of the parameters gave us the best fit to the experimental rapidity distribution for a particular hadron.

4 Results

We fitted three distributions of the BRAHMS experimental data: the $p - \bar{p}$, $\bar{p}/p$ and π^+ rapidity distributions to the model with the parameters, a, b and σ. The c++ code for the project is attached in appendix A.

4.1 Pion Data

We fitted the pion distribution first since it was the most sensetive to σ and the least sensetive to μ_B. This is because of the very large numbers of pions that are produced and the fact that they're not baryons. We also calculated the number of pions in the fireball from the integration of the curve shown below and found: $N_{\pi^+} = 1698$.

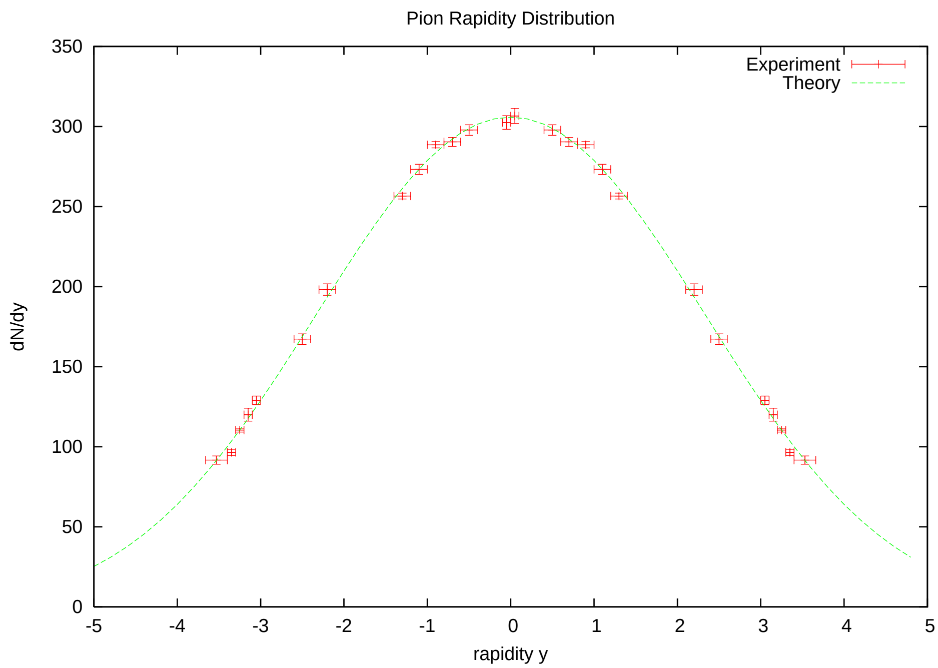

Figure 2: Fitted rapidity distribution to the π^+ BRAHMS data

The fit produced the following values for the parameters:

- $\sigma = 2.2236 \pm 0.0096$
- $a = 16.4466 \pm 0.0468$ MeV

- $b = 11.4330 \pm 1.2351$ MeV

4.2 Proton - Anti-Proton Data

The net proton distribution was also fitted. It was found to be a lot less sensitive to σ than the π^+ data. The graph of the results are presented below. The integration of the curve gives us that $N_{p-\bar{p}} = 98$. We know that from the initial conditions that this should be about twice the number of protons in a gold nucleus, i.e. 158.

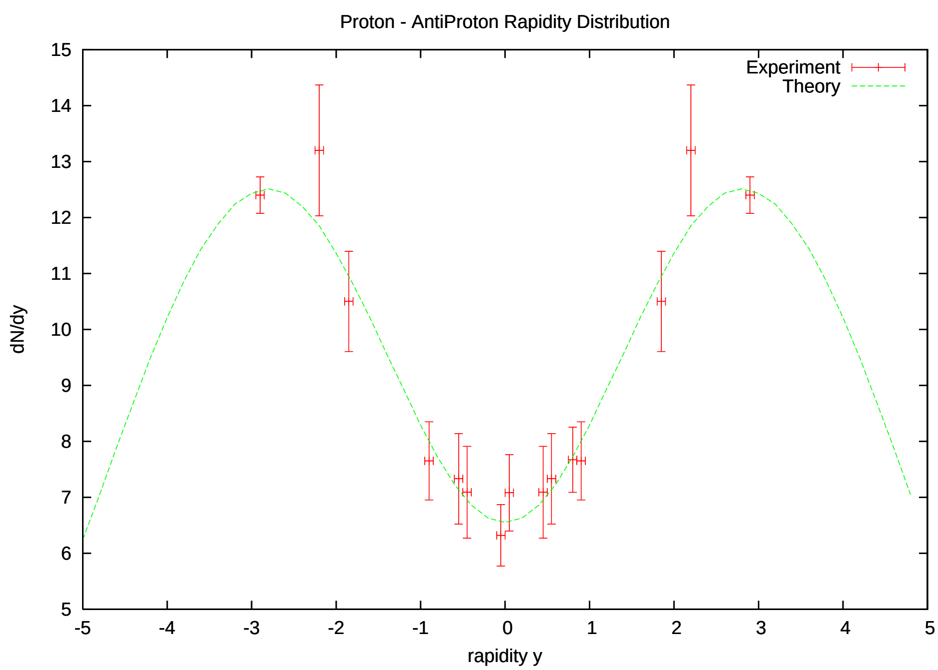

Figure 3: Fitted rapidity distribution to the $p - \bar{p}$ BRAHMS data

The fit to the net proton data produced:

- $\sigma = 2.3071 \pm 0.2521$
- $a = 25.0535 \pm 3.7264$ MeV
- $b = 10.9040 \pm 1.1880$ MeV

4.3 Anti-Proton / Proton Data

The fit to the ratio of anti-protons to protons was also done. The results are presented below:

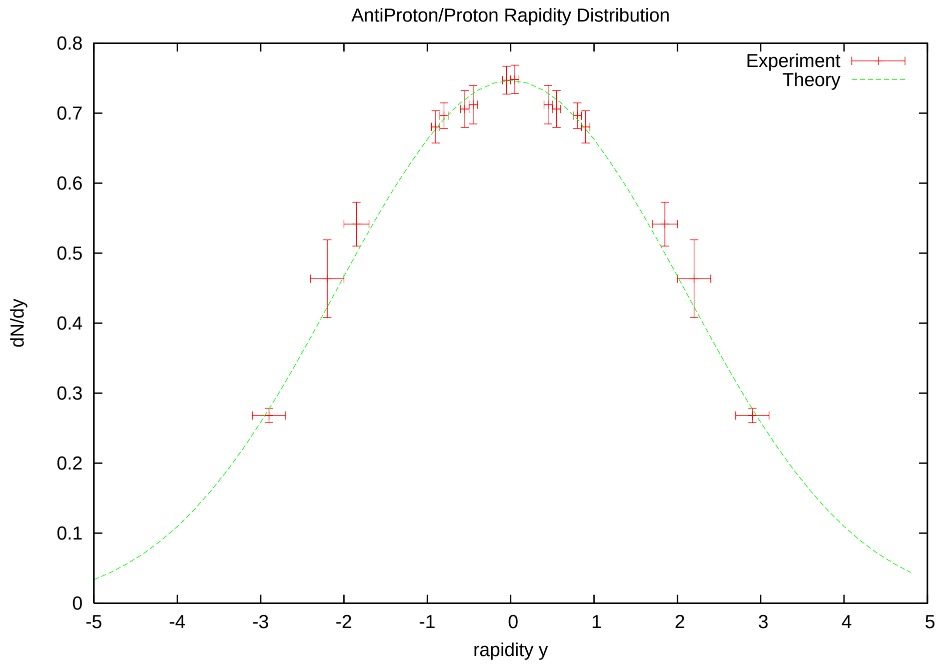

Figure 4: Fitted rapidity distribution to the $\bar{p}/p$ BRAHMS data

The values for the fitted parameters were:

- $\sigma = 2.0887 \pm 0.5080$
- $a = 23.8008 \pm 0.0180$ MeV
- $b = 11.2428 \pm 0.0080$ MeV

Combining these results produce the plots shown in figure (5).

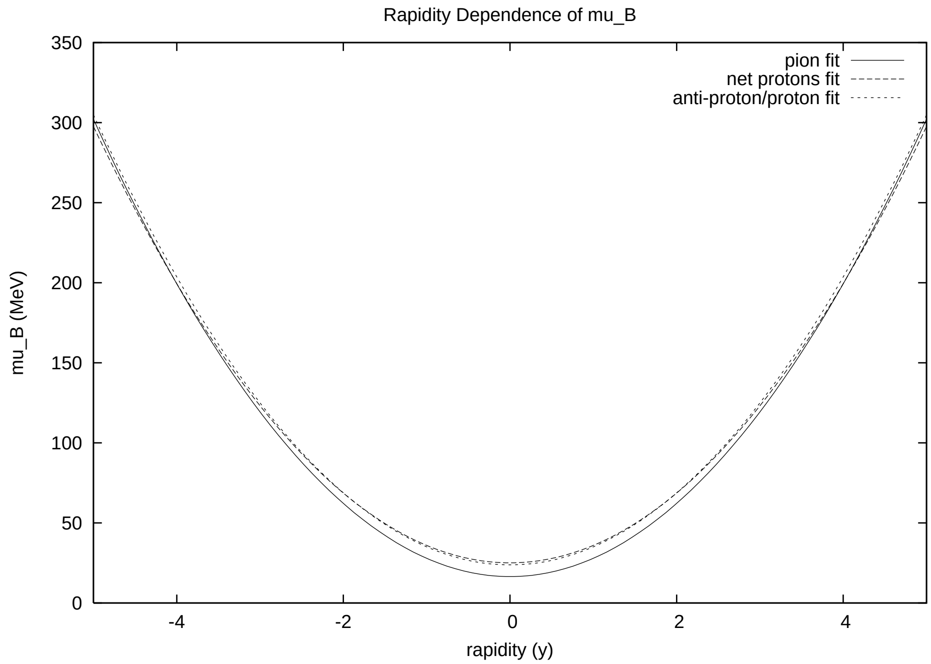

Figure 5: The resutls for the baryon chemical potential on rapidity. The dependence of the other thermal parameters are determined in turn by their dependence on μ_B shown in figure (1)

5 Conclusion

As can be seen from figures (2), (3) and (4), the model produced results that fit the data very well. The summary of the parameters from the fits are given in the table below. The insensitivity of the pion data to the baryon chemical potential can be seen from low value of a along with the small error on it. The a-values produced by the other two distributions are in agreement with each other, but not with the pion value. Both the other parameters, σ and b, have agreement amongst the three distributions.

Table 1: Comparison of fitted parameters

Distribution	σ		a [MeV]		b [MeV]	
	Value	Error	Value	Error	Value	Error
Net Protons	2.3071	0.2521	25.0535	3.7264	10.9040	1.1880
Proton Ratio	2.0887	0.5080	23.8008	0.0180	11.2428	0.0080
Pion	2.2236	0.0096	16.4466	0.0468	11.4330	1.2351

From the large errors on σ in the net proton and proton ratio distributions (11% and 24%) we can see that they are fairly insensitive to σ. The accurate fits to the experimental data show that the parameterisation of the rapidity dependence of μ_B used works well at the high RHIC energies and should thus also do so for the even higher LHC energies.

To proceed further, more experimental data should be used to garner more accurate parameters and to test the model properly for more energies. It may also be worth looking to the incorporation of the tranverse flow into the description, but this would be contrary to trying to find a simple description of the situation. Other parameterisations of the rapidity dependece could also be explored as well as different fireball distribuions along the rapidity axis.

I would like to thank Professor Jean Cleymans with his help in this project and for the oppertunity to work on his research.

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